Imaging with Ambient Noise II

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Outline: Imaging with ambient noise

1. Recovering the Green's function from cross correlations of recorded noisy signals. Using the Kirchhoff-Helmholtz identity
2. Using stationary phase to understand the role of directivity in noise sources. Background velocity estimation.
3. Imaging of reflectors with daylight illumination or with backlight illumination generated by noise sources.
4. Using fourth order cross correlations for imaging.

The empirical cross correlation

The empirical cross correlation of the signals recorded at $\mathbf{x}_1$ and $\mathbf{x}_2$ for an integration time T is

$$C_T(\tau, \mathbf{x}_1, \mathbf{x}_2) = \frac{1}{T} \int_0^T u(t, \mathbf{x}_1) u(t + \tau, \mathbf{x}_2) dt \,. \tag{1}$$

Here $u(t, \mathbf{x}_j)$ is the signal measured at $\mathbf{x}_j$ and T is chosen appropriately (a non-trivial statistical signal processing issue in applications).

The model GF calculation

The solution $u(t, \mathbf{x})$ of the wave equation in an inhomogeneous medium:

$$\frac{1}{c^2(\mathbf{x})}\frac{\partial^2 u}{\partial t^2} - \Delta_{\mathbf{x}} u = n^{\epsilon}(t, \mathbf{x}) \,. \tag{2}$$

The term $n^{\epsilon}(t, \mathbf{x})$ models a random distribution of noise sources. It is a zero-mean stationary (in time) Gaussian process with autocorrelation function

$$\langle n^{\epsilon}(t_1, \mathbf{y}_1) n^{\epsilon}(t_2, \mathbf{y}_2) \rangle = F^{\epsilon}(t_2 - t_1)\theta(\mathbf{y}_1)\delta(\mathbf{y}_1 - \mathbf{y}_2) \,. \tag{3}$$

We assume that the decoherence time of the noise sources is much smaller than typical travel times between sensors. If we denote with ϵ the (small) ratio of these two time scales, we can then write the time correlation function F^{ϵ} in the form

$$F^{\epsilon}(t_2 - t_1) = F\Big(\frac{t_2 - t_1}{\epsilon}\Big) \,, \tag{4}$$

where t_1 and t_2 are scaled relative to typical inter-sensor travel times.

The model GF calculation continued

The stationary solution of the wave equation has the integral representation

$$
\begin{aligned}
u(t,\mathbf{x}) &= \int\int_{-\infty}^{t} n^{\epsilon}(s,\mathbf{y})G(t-s,\mathbf{x},\mathbf{y})ds d\mathbf{y} \\
&= \int\int n^{\epsilon}(t-s,\mathbf{y})G(s,\mathbf{x},\mathbf{y})ds d\mathbf{y}\,, \qquad (5)
\end{aligned}
$$

where $G(t,\mathbf{x},\mathbf{y})$ is the time-dependent Green's function. It is the fundamental solution of the wave equation

$$
\frac{1}{c^2(\mathbf{x})}\frac{\partial^2 G}{\partial t^2} - \Delta_{\mathbf{x}} G = \delta(t)\delta(\mathbf{x}-\mathbf{y})\,, \qquad (6)
$$

starting from $G(0,\mathbf{x},\mathbf{y}) = \partial_t G(0,\mathbf{x},\mathbf{y}) = 0$ (and continued on the negative time axis by $G(t,\mathbf{x},\mathbf{y}) = 0\ \forall t \leqslant 0$).

The model GF calculation continued

The empirical cross correlation is a statistically stable quantity: For a large integration time T, C_T is independent of the realization of the noise sources.

1. The expectation of C_T (with respect to the distribution of the sources) is independent of T:

$$\langle C_T(\tau, \mathbf{x}_1, \mathbf{x}_2) \rangle = C^{(1)}(\tau, \mathbf{x}_1, \mathbf{x}_2)\,, \tag{7}$$

where $C^{(1)}$ is given by

$$C^{(1)}(\tau, \mathbf{x}_1, \mathbf{x}_2) = \int d\mathbf{y} \int ds ds' G(s, \mathbf{x}_1, \mathbf{y}) G(\tau + s + s', \mathbf{x}_2, \mathbf{y}) F^{\epsilon}(s') \theta(\mathbf{y})\,, \tag{8}$$

or equivalently in the frequency domain by

$$C^{(1)}(\tau, \mathbf{x}_1, \mathbf{x}_2) = \int d\mathbf{y} \int d\omega \overline{\hat{G}}(\omega, \mathbf{x}_1, \mathbf{y}) \hat{G}(\omega, \mathbf{x}_2, \mathbf{y}) \hat{F}^{\epsilon}(\omega) e^{-i\omega\tau} \theta(\mathbf{y})\,. \tag{9}$$

The basic GF calculation continued

2. The empirical cross correlation C_T is a self-averaging quantity:

$$C_T(\tau, \mathbf{x}_1, \mathbf{x}_2) \stackrel{T\to\infty}{\longrightarrow} C^{(1)}(\tau, \mathbf{x}_1, \mathbf{x}_2)\,, \tag{10}$$

in probability with respect to the distribution of the sources. More precisely, the fluctuations of C_T around its mean value $C^{(1)}$ are of order $T^{-1/2}$ for T large compared to the decoherence time of the sources. Here we assume that spatial covariance of the noise sources has the form

$$\Gamma(\mathbf{y}_1, \mathbf{y}_2) = \theta(\mathbf{y}_1)\delta(\mathbf{y}_1 - \mathbf{y}_2)\,, \tag{11}$$

That is, we have spatially uncorrelated noise sources.

The basic GF calculation continued

When the medium is homogeneous with background velocity c_0 and the source distribution extends over all space, i.e. $\theta \equiv 1$, the signal amplitude diverges because the contributions from noise sources far away from the sensors are not damped. So we consider the solution u of the damped wave equation:

$$\frac{1}{c_0^2}\Big(\frac{1}{T_a}+\frac{\partial}{\partial t}\Big)^2 u - \Delta_{\mathbf{x}} u = n^{\epsilon}(t,\mathbf{x})\,. \tag{12}$$

In a three-dimensional open medium with dissipation and if the source distribution extends over all space $\theta \equiv 1$, then

$$\frac{\partial}{\partial \tau}C^{(1)}(\tau,\mathbf{x}_1,\mathbf{x}_2) = -\frac{c_0^2 T_a}{4} e^{-\frac{|\mathbf{x}_1-\mathbf{x}_2|}{c_0 T_a}}\Big[F^{\epsilon} * G(\tau,\mathbf{x}_1,\mathbf{x}_2) - F^{\epsilon} * G(-\tau,\mathbf{x}_1,\mathbf{x}_2)\Big]\,, \tag{13}$$

where $*$ stands for the convolution in τ and G is the Green's function of the homogeneous medium without dissipation:

$$G(t,\mathbf{x}_1,\mathbf{x}_2) = \frac{1}{4\pi|\mathbf{x}_1-\mathbf{x}_2|}\delta\Big(t - \frac{|\mathbf{x}_1-\mathbf{x}_2|}{c_0}\Big)\,.$$

The basic GF calculation continued

If the decoherence time of the sources is much shorter than the travel time (i.e., $\epsilon \ll 1$), then F^ϵ behaves like a Dirac distribution and we have

$$\frac{\partial}{\partial\tau} C^{(1)}(\tau, \mathbf{x}_1, \mathbf{x}_2) \simeq e^{-\frac{|\mathbf{x}_1 - \mathbf{x}_2|}{c_0 T_a}} \Big[G(\tau, \mathbf{x}_1, \mathbf{x}_2) - G(-\tau, \mathbf{x}_1, \mathbf{x}_2) \Big] ,$$

up to a multiplicative constant.

It is therefore possible to estimate the travel time $\tau(\mathbf{x}_1, \mathbf{x}_2) = |\mathbf{x}_1 - \mathbf{x}_2|/c_0$ between $\mathbf{x}_1$ and $\mathbf{x}_2$ from the cross correlation, with an accuracy of the order of the decoherence time of the noise sources.

The basic GF calculation some details

The Green's function of the homogeneous medium with dissipation is:

$$G_a(t,\mathbf{x}_1,\mathbf{x}_2) = G(t,\mathbf{x}_1,\mathbf{x}_2)e^{-\frac{t}{T_a}}\,.$$

The cross correlation function is given by (8):

$$C^{(1)}(\tau,\mathbf{x}_1,\mathbf{x}_2) = \int d\mathbf{y}\int ds ds' G_a(s,\mathbf{x}_1,\mathbf{y})G_a(\tau+s+s',\mathbf{x}_2,\mathbf{y})F^{\epsilon}(s')\,.$$

Integrating in s and s' gives

$$C^{(1)}(\tau,\mathbf{x}_1,\mathbf{x}_2) = \int \frac{d\mathbf{y}}{16\pi^2|\mathbf{x}_1-\mathbf{y}|\,|\mathbf{x}_2-\mathbf{y}|}e^{-\frac{|\mathbf{x}_1-\mathbf{y}|+|\mathbf{x}_2-\mathbf{y}|}{c_0T_a}}F^{\epsilon}\Big(\tau-\frac{|\mathbf{x}_1-\mathbf{y}|-|\mathbf{x}_2-\mathbf{y}|}{c_0}\Big)$$

The basic GF calculation

We parameterize the locations of the sensors by $\mathbf{x}_1 = (h, 0, 0)$ and $\mathbf{x}_2 = (-h, 0, 0)$, where $h > 0$, and we use the change of variables for $\mathbf{y} = (x, y, z)$:

$$\begin{cases} x = h \sin\theta \cosh\phi \,, & \phi \in (0, \infty) \,, \\ y = h \cos\theta \sinh\phi \cos\psi \,, & \theta \in (-\pi/2, \pi/2) \,, \\ z = h \cos\theta \sinh\phi \sin\psi \,, & \psi \in (0, 2\pi) \,, \end{cases}$$

whose Jacobian is $J = h^3 \cos\theta \sinh\phi (\cosh^2\psi - \sin^2\theta)$. Using the fact that $|\mathbf{x}_1 - \mathbf{y}| = h(\cosh\phi - \sin\theta)$ and $|\mathbf{x}_2 - \mathbf{y}| = h(\cosh\phi + \sin\theta)$, we get

$$C^{(1)}(\tau, \mathbf{x}_1, \mathbf{x}_2) = \frac{h}{8\pi} \int_0^\infty d\phi \sinh\phi \int_{-\pi/2}^{\pi/2} d\theta \cos\theta e^{-\frac{2h\cosh\phi}{c_0 T_a}} F^\epsilon\Big(\tau + \frac{2h\sin\theta}{c_0}\Big) \,.$$

Continued

After the new change of variables $u = h \cosh \phi$ and $s = (2h/c_0) \sin \theta$, we obtain

$$C^{(1)}(\tau, \mathbf{x}_1, \mathbf{x}_2) = \frac{c_0^2 T_a}{32\pi h} e^{-\frac{2h}{c_0 T_a}} \int_{-2h/c_0}^{2h/c_0} F^{\epsilon}(\tau + s) ds \, .$$

By differentiating in τ, we get

$$\frac{\partial}{\partial \tau} C^{(1)}(\tau, \mathbf{x}_1, \mathbf{x}_2) = \frac{c_0^2 T_a}{32\pi h} e^{-\frac{2h}{c_0 T_a}} \left[F^{\epsilon}\left(\tau + \frac{2h}{c_0}\right) - F^{\epsilon}\left(\tau - \frac{2h}{c_0}\right) \right],$$

which is the desired result since $|\mathbf{x}_1 - \mathbf{x}_2| = 2h$.

The Kirchhoff-Helmholtz identity

Let us assume that the medium is homogeneous with background velocity c_e outside the ball $B(\mathbf{0}, D)$ with center $\mathbf{0}$ and radius D. Then, for any $\mathbf{x}_1, \mathbf{x}_2 \in B(\mathbf{0}, D)$ we have for $L \gg D$:

$$\hat{G}(\omega, \mathbf{x}_1, \mathbf{x}_2) - \overline{\hat{G}}(\omega, \mathbf{x}_1, \mathbf{x}_2) = \frac{2i\omega}{c_e} \int_{\partial B(\mathbf{0}, L)} \overline{\hat{G}}(\omega, \mathbf{x}_1, \mathbf{y}) \hat{G}(\omega, \mathbf{x}_2, \mathbf{y}) d\sigma(\mathbf{y}) . \tag{14}$$

Key idea here: Apply the Sommerfeld radiation condition to Green's identity for the Green's functions.
Note that it is important that the medium be homogeneous in the exterior of the ball $B(\mathbf{0}, D)$ in order be able to use the radiation condition.

Application of the KH identity

We assume that:
1. The medium is homogeneous with background velocity c_e outside the ball $B(\mathbf{0}, D)$ with center $\mathbf{0}$ and radius D.
2. The sources are localized with a uniform density on the sphere $\partial B(\mathbf{0}, L)$ with center $\mathbf{0}$ and radius L.
If $L \gg D$, then for any $\mathbf{x}_1, \mathbf{x}_2 \in B(\mathbf{0}, D)$

$$\frac{\partial}{\partial \tau} C^{(1)}(\tau, \mathbf{x}_1, \mathbf{x}_2) = -F^\epsilon * G(\tau, \mathbf{x}_1, \mathbf{x}_2) + F^\epsilon * G(-\tau, \mathbf{x}_1, \mathbf{x}_2)\,, \qquad (15)$$

up to a multiplicative factor. Here $*$ stands for convolution in τ.

More on the KH identity

The second Green's identity applied to the Greens functions at $\mathbf{x}_1$ and $\mathbf{x}_2$ has the form

$$\int_{\partial B(\mathbf{0},L)} \mathbf{n} \cdot \left[\overline{\hat{G}}(\omega, \mathbf{y}, \mathbf{x}_1) \nabla_{\mathbf{y}} \hat{G}(\omega, \mathbf{y}, \mathbf{x}_2) - \hat{G}(\omega, \mathbf{y}, \mathbf{x}_2) \nabla_{\mathbf{y}} \overline{\hat{G}}(\omega, \mathbf{y}, \mathbf{x}_1)\right] d\sigma(\mathbf{y})$$
$$= \hat{G}(\omega, \mathbf{x}_1, \mathbf{x}_2) - \overline{\hat{G}}(\omega, \mathbf{x}_1, \mathbf{x}_2)\,,$$

where $\mathbf{n}$ is the unit outward normal to the ball $B(\mathbf{0}, L)$, which is $\mathbf{n} = \mathbf{y}/|\mathbf{y}|$.

Green's identity + Radiation condition = KH identity

The Green's function also satisfies the Sommerfeld radiation condition

$$\lim_{|\mathbf{y}|\to\infty} |\mathbf{y}|^{\frac{d-1}{2}} \Big(\frac{\mathbf{y}}{|\mathbf{y}|} \cdot \nabla_{\mathbf{y}} - \, i \frac{\omega}{c_e} \Big) \hat{G}(\omega, \mathbf{y}, \mathbf{x}_1) = 0 \,,$$

uniformly in all directions $\mathbf{y}/|\mathbf{y}|$. Under the conditions stated in the proposition, we can substitute $i(\omega/c_e)\hat{G}(\omega, \mathbf{y}, \mathbf{x}_2)$ for $\mathbf{n} \cdot \nabla_{\mathbf{y}} \hat{G}(\omega, \mathbf{y}, \mathbf{x}_2)$ in the surface integral over $\partial B(\mathbf{0}, L)$, and $-i(\omega/c_e)\overline{\hat{G}}(\omega, \mathbf{y}, \mathbf{x}_1)$ for $\mathbf{n} \cdot \nabla_{\mathbf{y}} \overline{\hat{G}}(\omega, \mathbf{y}, \mathbf{x}_1)$.

Using stationary phase

What happens when the noise sources are not "everywhere"?
This is a very important issue in practice, in seismology for example where there is a lot of interest in using cross correlation methods for velocity estimation.
The outgoing time-harmonic Green's function $\hat{G}_0$ is the solution of

$$\Delta_{\mathbf{x}} \hat{G}_0(\omega, \mathbf{x}, \mathbf{y}) + \frac{\omega^2}{c_0^2(\mathbf{x})} \hat{G}_0(\omega, \mathbf{x}, \mathbf{y}) = -\delta(\mathbf{x} - \mathbf{y}), \tag{16}$$

along with the radiation condition at infinity. When the background is homogeneous with constant wave speed c_0 and wavenumber $k = \omega/c_0$, then

$$\hat{G}_0(\omega, \mathbf{x}, \mathbf{y}) = \frac{e^{ik|\mathbf{y}-\mathbf{x}|}}{4\pi|\mathbf{y} - \mathbf{x}|} \tag{17}$$

in three-dimensional space.

Using stationary phase, continued

For a general smoothly varying background with propagation speed $c_0(\mathbf{x})$, the high-frequency behavior of the Green's function is also related to the travel time and it is given by the WKB (Wentzel-Kramers-Brillouin) approximation

$$\hat{G}_0\Big(\frac{\omega}{\epsilon}, \mathbf{x}, \mathbf{y}\Big) \sim a(\mathbf{x}, \mathbf{y}) e^{i\frac{\omega}{\epsilon}\tau(\mathbf{x},\mathbf{y})}, \tag{18}$$

which is valid when $\epsilon \ll 1$. Here the coefficients $a(\mathbf{x}, \mathbf{y})$ and $\tau(\mathbf{x}, \mathbf{y})$ are smooth except at $\mathbf{x} = \mathbf{y}$. The amplitude $a(\mathbf{x}, \mathbf{y})$ satisfies a transport equation and the travel time $\tau(\mathbf{x}, \mathbf{y})$ satisfies the eikonal equation. It is a symmetric function $\tau(\mathbf{x}, \mathbf{y}) = \tau(\mathbf{y}, \mathbf{x})$ and it can be obtained from Fermat's principle

$$\tau(\mathbf{x}, \mathbf{y}) = \inf\Big\{T \text{ s.t.} \exists\, (\mathbf{X}_t)_{t\in[0,T]} \in \mathcal{C}^1, \, \mathbf{X}_0 = \mathbf{x}, \, \mathbf{X}_T = \mathbf{y}, \, \Big|\frac{d\mathbf{X}_t}{dt}\Big| = c_0(\mathbf{X}_t)\Big\}. \tag{19}$$

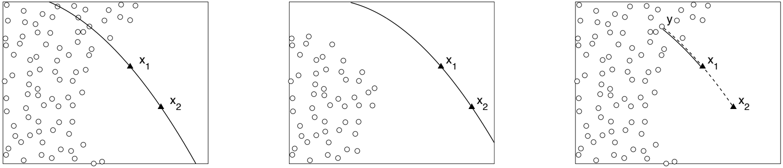
A curve $(\mathbf{X}_t)_{t\in[0,T]}$ that produces the minimum in (19) is a ray and it satisfies Hamilton's equations.

Using stationary phase, continued

We can now describe the behavior of the cross correlation function between $\mathbf{x}_1$ and $\mathbf{x}_2$ when ϵ is small, with and without directional energy flux from the sources.

As ϵ tend to zero, the cross correlation $C^{(1)}(\tau, \mathbf{x}_1, \mathbf{x}_2)$ has singular components if and only if the ray going through $\mathbf{x}_1$ and $\mathbf{x}_2$ reaches into the source region, that is, into the support of the function θ. In this case there are either one or two singular components at $\tau = \pm\tau(\mathbf{x}_1, \mathbf{x}_2)$. More precisely, any ray going from the source region to $\mathbf{x}_2$ and then to $\mathbf{x}_1$ gives rise to a singular component at $\tau = -\tau(\mathbf{x}_1, \mathbf{x}_2)$. Rays going from the source region to $\mathbf{x}_1$ and then to $\mathbf{x}_2$ give rise to a singular component at $\tau = \tau(\mathbf{x}_1, \mathbf{x}_2)$.

Using stationary phase, continued



If the ray going through $\mathbf{x}_1$ and $\mathbf{x}_2$ (solid line) enters into the source region (left figure), then the travel time can be estimated from the cross correlation. If this is not the case, then the cross correlation does not have a peak at the travel time (middle figure). Right figure: The main contribution to the singular components of the cross correlation is from pairs of ray segments issuing from a source $\mathbf{y}$ going to $\mathbf{x}_1$ and to $\mathbf{x}_2$, respectively (solid and dashed lines, respectively), and starting in the same direction.

Using stationary phase, continued

This proposition explains why travel time estimation is bad when the ray joining $\mathbf{x}_1$ and $\mathbf{x}_2$ is roughly perpendicular to the direction of the energy flux from the noise sources, as in the middle of the Figure.

The stationary phase contributions to the singular components of the cross correlation come from pairs of ray segments. The first ray goes from a source point to $\mathbf{x}_2$ and the second ray goes from the same source point and with the same initial angle to $\mathbf{x}_1$.

The singular component is then concentrated at the difference of the travel times between these two ray segments. In the configuration on the right in the Figure the contribution to the singular component is at $\tau = \tau(\mathbf{x}_1, \mathbf{x}_2)$.

Using stationary phase, continued

We first we use the WKB approximation (18) of the Green's function and obtain

$$C^{(1)}(\tau, \mathbf{x}_1, \mathbf{x}_2) = \frac{1}{2\pi} \int d\mathbf{y}\theta(\mathbf{y}) \int d\omega \hat{F}(\omega) \overline{a}(\mathbf{x}_1, \mathbf{y}) a(\mathbf{x}_2, \mathbf{y}) e^{i \frac{\omega}{\epsilon} \mathcal{T}(\mathbf{y})} ,$$

where the rapid phase is

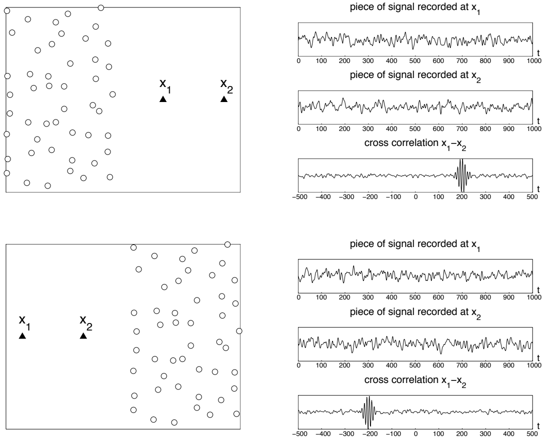
$$\omega \mathcal{T}(\mathbf{y}) = \omega[\tau(\mathbf{x}_2, \mathbf{y}) - \tau(\mathbf{x}_1, \mathbf{y}) - \tau] \; . \tag{20}$$

and the $(\omega \mathbf{y})$ stationary points satisfy

$$\nabla_\omega \omega \mathcal{T}(\mathbf{y}) = 0 \; , \quad \nabla_\mathbf{y} \omega \mathcal{T}(\mathbf{y}) = 0$$

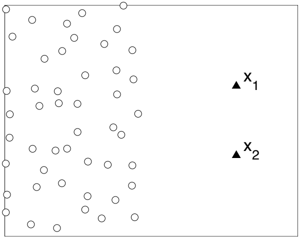
Now we use a standard multi-dimensional stationary phase approximation.

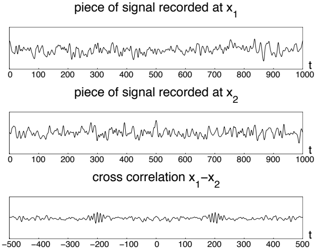
Noise signal cross correlations



When the distribution of noise sources is spatially localized then the cross correlation function is not symmetric

Noise signal cross correlations, continued





When the distribution of noise sources is spatially localized then the coherent part of the cross correlation function can be difficult or even impossible to distinguish if the axis formed by the two sensors is perpendicular to the main direction of energy flux from the noise sources.

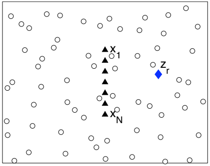
Passive sensor imaging of reflectors with ambient noise

We have shown how to estimate inter-sensor travel times using cross correlations of noise signals. And there are at least two ways this information can be used in applications.

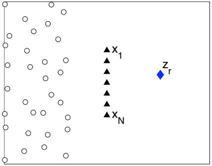
What about imaging <u>reflectors</u> in the medium?

Start with homogeneous media. Then comment on the much more important case of scattering media.

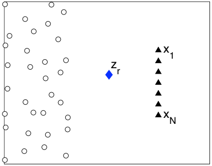
Imaging with cross correlations: Different illuminations



Surround light



Daylight



Backlight

Left figure: The noise sources are distributed throughout the medium, which is the surround light imaging configuration. Middle figure: the sensors $(\mathbf{x}_j)_{j=1,\dots,N}$ are between the noise sources and the reflector $\mathbf{z}_r$, which is the daylight imaging configuration. Right figure: the reflector is between the noise sources and the sensors, which is the backlight imaging configuration.

Use stationary phase again

When scattering from the reflector is included in the signal model, the cross correlations have additional peaks. They come from additional stationary points.

In the case of daylight illumination the additional peaks are at the sensor-to-reflector sum travel times. But the peaks are weak. We must use differential cross correlations, or coda cross correlations.

In the case of backlight illumination the additional peaks are at the sensor-to-reflector difference travel times. But the peaks are weak. We must use differential cross correlations. Coda cross correlations cannot work!

Migration imaging functional

Daylight (D and +) and Backlight (B and -), differential correlation imaging functionals. We use "symmetrized" correlations at positive lags only.

$$\mathcal{J}^{\mathsf{D,B}}(\mathbf{z}^S) \quad = \quad \sum_{j,l=1}^{N} \left[C^{\mathsf{sym}} - C_0^{\mathsf{sym}}\right]\big(\tau(\mathbf{z}^S, \mathbf{x}_l) \pm \tau(\mathbf{z}^S, \mathbf{x}_j), \mathbf{x}_j, \mathbf{x}_l\big)\,, \qquad (21)$$

Analysis in: Josselin Garnier and George Papanicolaou, Passive Sensor Imaging Using Cross Correlations of Noisy Signals in a Scattering Medium, SIAM J. IMAGING SCIENCES 2009 Vol. 2, No. 2, pp. 396437

Why migration imaging?

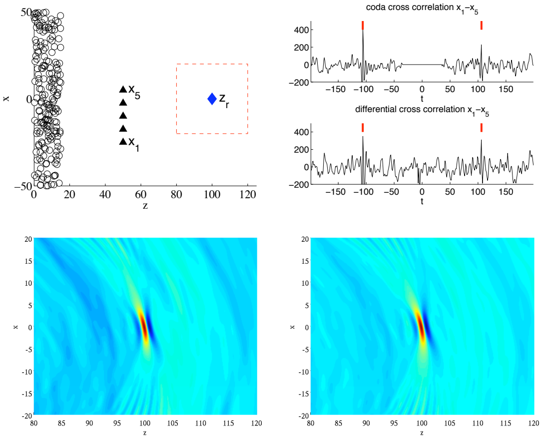
Interpretation of migration as time reversal imaging

Interpretation as (approximate) least squares imaging

Cross range resolution: $\frac{\lambda L}{a}$ where L is the range and a is the array size

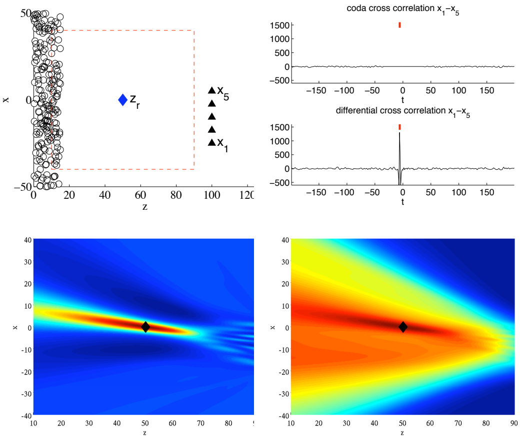
Range resolution: $\frac{c}{B}$ where B is the bandwidth

Daylight passive sensor imaging



Daylight imaging with passive sensors (triangles on top left). The reflector (diamond) is illuminated by noise sources (circles). Coda and differential correlations at the ends of the array, top right. Imaging with differential cross correlations at bottom left. With coda at bottom right.

Backlight passive sensor imaging

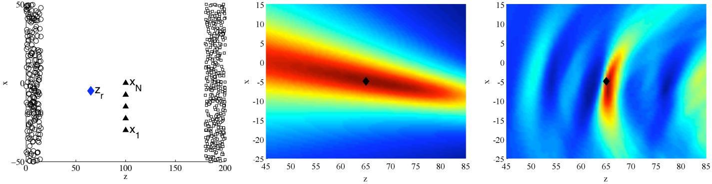


Backlight migration imaging with five passive sensors (triangles on the top left figure). The reflector (diamond) is at $(x_r = 0, z_r = 50)$, illuminated by noise sources (circles). The coda correlation and the differential correlation of the signals for the two sensors at the ends of the array is in top right figure.

Noise source imaging in scattering media

- What if the background is not smoothly varying but scattering?
- A scattering environment plays a dual role: (a) it helps diversify the noise sources, but (b) it also blurs the image. Can sort this with the transport mean free path.
- Feasibility study of the potential imaging capabilities of passive sensor networks in noisy environments is being done (it is difficult).

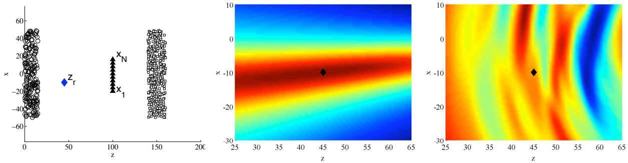
Passive sensor imaging I



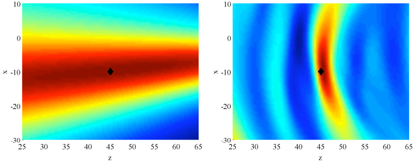
a) Configuration **b)** $\mathcal{J}^{\mathrm{B}}$ **c)** $\mathcal{J}^{\mathrm{D}}$

Passive sensor imaging using the differential cross correlation technique in a scattering medium. The configuration is plotted in Figure a: the circles are the noise sources, the squares are the scatterers. Figure b plots the image obtained with the backlight imaging functional. Figure c plots the image obtained with the daylight imaging functional.

Passive sensor imaging II



a) Configuration **b)** $\mathcal{J}^{\mathrm{B}}$ with $\mathrm{C}^{(1)}$ **c)** $\mathcal{J}^{\mathrm{D}}$ with $\mathrm{C}^{(1)}$



d) $\mathcal{J}^{\mathrm{B}}$ with $\mathrm{C}^{(3)}$ **e)** $\mathcal{J}^{\mathrm{D}}$ with $\mathrm{C}^{(3)}$

Configuration Figure a: circles are noise sources, squares are scatterers. Figure b from the backlight imaging functional and the cross correlation $\mathrm{C}^{(1)}$. Figure c from the daylight i.f. and the cross correlation $\mathrm{C}^{(1)}$. Figure d from the backlight i.f. and the coda cross correlation $\mathrm{C}^{(3)}$. Figure e from the daylight i.f. and the coda cross correlation $\mathrm{C}^{(3)}$.

Concluding remarks

- Passive sensor imaging with noise sources and in scattering media has huge potential, both theoretical and applied. It is mathematically challenging and draws on many related research areas (wave propagation, random media, statistical signal processing, optimization, ...).
- Theoretical issues, resolution theory for example, can often be reduced to already established results in active sensor imaging, both for distributed sensors and for arrays.
- Using interferometric methods (CINT) for passive sensor imaging is a very promising and is now under study.