1. Consider the series

$$\sum \frac{1}{n},$$

the sum is taken over those natural numbers n that contain no 9 in their decimal representations. Does the series converge?

2. Prove the following equality for all natural numbers n:

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{2n-1}-\frac{1}{2n}=\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}.$$

3. In a room there are 10 people, each of which has age between 1 and 60 (ages are only integers). Prove that among them there are two groups of people with no commom person, the sum of whose ages is the same.

4. Let f be a continuously differentiable complex function on $[0,1]$ such that

$$f(0)=f(1)=0 \quad \text{and} \quad \int_0^1 f(t)dt=1.$$

Prove that there exists a point s in $[0,1]$ such that $|f'(s)|\geq 4$.

5. A hare and an elephant start from the same point and move along a straight road in the same direction, each at a constant speed. After completing 400 leaps, the hare turns around. At this moment the elephant starts counting his own steps. Given that the elephant has counted 70 steps by the moment he met the hare on his way back and assuming that all steps of the elephant are of the same length and all leaps of the hare are of the same length, which is longer: 1 step of the elephant or one leap of the hare?

6. There are 34 tussocks in the swamp. A frog can leap from one tussock to another but doesn't want to swim in the swamp. It is known that from each tussock the frog can reach at least five other tussocks in one leap. Prove that if the frog can reach a certain tussock from the tussock on which it sits without swimming, it can always do it in fewer than 15 leaps, but not necessarily in fewer than 14 leaps.